# Measuring how people learn how to plan

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#### Abstract

The human mind has an unparalleled ability to acquire complex cognitive skills, discover new strategies, and refine its ways of thinking and decision-making; these phenomena are collectively known as cognitive plasticity. One important manifestation of cognitive plasticity is learning to make better more far-sighted - decisions via planning. A serious obstacle to studying how people learn how to plan is that cognitive plasticity is even more difficult to observe than cognitive strategies are. To address this problem, we develop a computational microscope for measuring cognitive plasticity and validate it on simulated and empirical data. Our approach employs a process tracing paradigm recording signatures of human planning and how they change over time. We then invert a generative model of the recorded changes to infer the underlying cognitive plasticity. Our computational microscope measures cognitive plasticity significantly more accurately than simpler approaches, and it correctly detected the effect of an external manipulation known to promote cognitive plasticity. We illustrate how computational microscopes can be used to gain new insights into the time course of metacognitive learning and to test theories of cognitive development and hypotheses about the nature of cognitive plasticity. Future work will leverage our computational microscope to reverse-engineer the learning mechanisms enabling people to acquire complex cognitive skills such as planning and problem solving.

**Keywords:** cognitive plasticity; planning; decision-making; process-tracing; statistical methods

#### Introduction

One of the most remarkable features of the human mind is its ability to continuously improve itself. As helpless babies develop into mature adults, their brains do not only acquire impressive perceptual and sensory-motor skills and knowledge about the world but they also learn to think, to make better decisions, to learn, and to monitor and adaptively regulate themselves. These phenomena are collectively known as *cognitive plasticity*. Just like the acquisition of perceptual skills (Hubel & Wiesel, 1970), the acquisition of cognitive skills requires specific experiences and practice (van Lehn, 1996; Ericsson, Krampe, & Tesch-Römer, 1993).

Despite initial research on how people acquire cognitive skills (van Lehn, 1996; Shrager & Siegler, 1998; Krueger, Lieder, & Griffiths, 2017), the underlying learning mechanisms are still largely unknown. Reverse-engineering how people learn how to think and how to decide is very challenging because we can neither observe people's cognitive strategies, nor how they change with experience – let alone

the underlying learning mechanisms. Instead, cognitive plasticity has to be inferred from observable changes in behavior. This is difficult because each observed behavior could have been generated by many possible cognitive mechanisms. This problem is pertinent to all areas of cognition. As a first step towards a more general solution, we develop a computational microscope for measuring how people learn how to plan.

Our approach combines a recently developed processtracing paradigm that renders people's behavior highly diagnostic of their planning strategies with probabilistic models of planning and learning that constrain the space of potential cognitive mechanisms and exploit temporal dependencies among subsequent planning strategies. Critically, our measurement model can be inverted to infer the sequence of people's planning strategies from the clicks they make in the process tracing paradigm.

Our computational microscope makes it possible to observe how people's planning strategies change from each decision to the next. This sheds new light on the time course and the nature of metacognitive learning. Future work will reverse-engineer the learning mechanisms that generate the cognitive plasticity our computational microscope is bringing to light.

The plan for this paper is as follows: we start by developing a computational method for measuring cognitive plasticity. Next, we validate it on synthetic data and human data. We then illustrate the utility of our computational microscope by measuring the time course of how people learn how to plan, characterizing the revealed learning trajectories, and testing theories of cognitive development and cognitive plasticity. In closing, we discuss the implications of our findings and directions for future work.

#### Methods

### Process-tracing using the Mouselab-MDP paradigm

Planning, like all cognitive processes, cannot be observed directly but has to be inferred from observable behavior. This is generally an ill-posed problem. To address this challenge, researchers have developed *process-tracing* methods that elicit and record behavioral signatures of latent cognitive processes; for instance decision strategies can be traced by recording the order in which people inspect the payoffs of

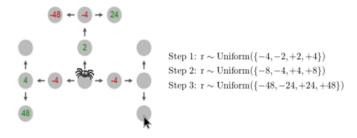


Figure 1: Illustration of the Mouselab-MDP paradigm. Rewards are revealed by clicking, prior to selecting a path with the arrow keys. The distribution of rewards underlying each node at a given step is shown on the right.

different gambles (Payne, Bettman, & Johnson, 1993). While these behavioral signatures are still indirect measures of cognitive processes, they do provide additional information about what the underlying cognitive strategy might be.

Here, we employ a process-tracing paradigm that externalizes people's beliefs and planning operations as observable states and actions (Callaway, Lieder, Krueger, & Griffiths, 2017; Callaway et al., 2018). Inspired by the Mouselab paradigm (Payne et al., 1993), the Mouselab-MDP paradigm uses people's mouse-clicking as a window into their planning.

The Mouselab-MDP paradigm illustrated in Figure 1 presents a series of route planning problems where each location (the gray circles), harbors a gain or loss. These potential gains and losses are initially occluded, corresponding to a highly uncertain belief state. The participant can reveal each location's reward by clicking on it and paying a fee. This is similar to looking at a map to plan a road trip. Clicking on a circle corresponds to thinking about a potential destination, evaluating how enjoyable it would be to go there, and adjusting one's assessment of candidate routes accordingly.

#### **Measurement model**

To construct a computational microscope for measuring cognitive plasticity, we develop hidden Markov models where the trial-by-trial sequence of peoples' cognitive strategies  $(S_1, S_2, \dots, S_{31})$  forms a Markov chain and the observed process tracing data constitutes the emissions. These models require methodological assumptions about i) how cognitive strategies manifest in process-tracing data, ii) the space of cognitive mechanisms that can be learned, and iii) the nature and amount of cognitive plasticity that might occur. The following paragraphs detail our assumptions about each of these three components in turn.

**Observation model.** To plan in the Mouselab-MDP paradigm participants have to gather information by making a sequence of clicks. Our observation model thus specifies the probability of of observing a sequence of clicks  $\mathbf{d_t}$  on trial t if the strategy was  $S_t$  (i.e.,  $P(\mathbf{d_t}|S_t)$ ).

To achieve this, we quantify each planning strategy's propensity to generate a click c (or stop collecting information) in belief state b by a weighted sum of the features  $f_1(b,c),\cdots,f_{29}(b,c)$ . The belief state encodes observed rewards. The features describe the click c relative to this information (e.g., by the value of the largest reward that can be collected from the inspected location) and in terms of the action it gathers information about (e.g., whether it pertains to the first, second, or third step)<sup>1</sup>. The features and weights jointly determine the strategy's propensity to make click c in belief state b according to

$$P(\mathbf{d_t}|S_t) = \prod_{i=1}^{|\mathbf{d_t}|} \frac{\exp\left(\frac{1}{\tau} \cdot \sum_{k=1}^{|w^{(S)}|} w_k^{(S)} \cdot f_k^{(S)}(c_{t,i}, b_{t,i})\right)}{\sum_{c \in C_{b_t}} \exp\left(\frac{1}{\tau} \cdot \sum_{k=1}^{|w^{(S)}|} w_k^{(S)} \cdot f_k^{(S)}(c, b_{t,i})\right)}, (1)$$

where  $d_{t,i}$  is the i<sup>th</sup> click the participant made on trial t (or the decision to stop clicking and take action), the decision temperature  $\tau$  was set to 0.5 to match the variability of people's click sequences, and  $w^{(S)}$  is the weight vector of strategy S.

**Space of cognitive mechanisms.** We formulated a set of 38 strategies  $(S)^1$  to describe the process tracing data from Lieder (2018): 76.7% of the click sequences were the most likely instantiation of one of the 38 strategies. The clicks of the remaining 23.3% of the sequences were, at worst, second most likely under the best fitting strategy. These strategies differ in how much information they consider (ranging from none to all), which information they focus on, and in the order in which they collect it (e.g., forward planning strategies prioritize immediate outcomes whereas goal-setting strategies focus on the potential final states).

Building on the observation model in Equation 1, we represent each strategy by a weight vector  $\mathbf{w} = (w_1, \dots, w_{29})$  that specifies the strategy's preference for more vs. less planning, considering immediate vs. long-term consequences, satisficing vs. maximizing, avoiding losses (cf. Huys et al., 2012), and other desiderata. These weights span a high-dimensional continuous space with many intermediate strategies and mixtures of strategies. Cognitive plasticity could be measured by tracking how those weights change over time. But this would be a very difficult ill-defined inference problem whose solution would depend on our somewhat arbitrary choice of features. As a first approximation, our method therefore simplifies the problem of measuring cognitive plasticity to inferring a time-series of discrete strategies.

To understand what types of strategies people use, we grouped our 38 strategies using hierarchical clustering. This requires measuring the similarity between strategies. Since the strategies are probabilistic, we defined the distance metric  $\Delta(s_1, s_2)$  between strategy  $s_1$  and  $s_2$  as the Jensen-Shannon divergence (Lin, 1991) between the distributions of click se-

<sup>&</sup>lt;sup>1</sup>A detailed description of the features and strategies is available at https://osf.io/y58d3/?view\_only=fa2f89de3aa04d4d87af3d050bbla64c

quences and belief states induced by strategies  $s_1$  and  $s_2$  respectively, that is

$$\Delta(s_1, s_2) = JS[p(\mathbf{d}|s_1), p(\mathbf{d}|s_2)], \qquad (2)$$

and approximate it using Monte-Carlo integration.

Applying Ward's hierarchical clustering method (Ward Jr, 1963) to the resulting distances suggested 11 types of planning strategies: acting impulsively without any planning, finding a goal and immediately moving towards it, inspecting both immediate and final outcomes (but no intermediate ones), overly frugal goal setting strategies, goal setting strategies that plan towards potential goals even when it is wasteful, exhaustive backward planning strategies that inspect all of the states, other far-sighted strategies that inspect all potential final states, forward-planning strategies similar to depthfirst search, forward-planning strategies similar to best-first search, strategies similar to breadth-first search, and strategies that focus on the course of action that has received the most consideration so far.

**Prior on strategy sequences.** Inferring a strategy from a single click sequence could be unreliable. Our method therefore exploits temporal dependencies between subsequent strategies to smooth out its inferences. Transitions from one strategy to the next can be grouped into three types: repetitions, gradual changes, and abrupt changes. While most neuroscientific and reinforcement-learning perspectives emphasize gradual learning (e.g., Hebb, 1949; Mercado III, 2008; Lieder, Shenhav, Musslick, & Griffiths, 2018), others suggest that animals change their strategy abruptly when they detect a change in the environment (Gershman, Blei, & Niv, 2010). Symbolic models and stage theories of cognitive development also assume abrupt changes (e.g., Piaget, 1971; Shrager & Siegler, 1998), and it seems plausible that both types of mechanisms might coexist. To accommodate these different perspectives, we consider three prior distributions on participants' trial-by-trial sequence of cognitive strategies.

The *gradual learning prior* in Equation 3 assumes that strategies changes gradually, that is

$$P(S_{t+1} = s | S_t, m_{\text{gradual}}) = \frac{\exp(-\frac{1}{\tau} \cdot \Delta(s, S_t))}{\sum_{s' \in \mathcal{S}} \exp(-\frac{1}{\tau} \cdot \Delta(s', S_t))}, \quad (3)$$

where S is the set of strategies, |S| is the number of strategies, and the temperature parameter  $\tau$  was set to achieve a 50% chance of a strategy change. By contrast, the *abrupt changes prior* in Equation 4 assumes that transitions are either repetitions or jumps.

$$P(S_{t+1} = s | S_t, m_{\text{abrupt}}) = p_{\text{stay}} \cdot \mathbb{I}(S_{t+1} = S_t) + (1 - p_{\text{stay}}) \cdot \frac{\mathbb{I}(s \neq S_t)}{|S| - 1},$$
(4)

Finally, the *mixed prior* in Equation 5 assumes that both types

of changes coexist.

$$P(S_{t+1} = s | S_t, m_{\text{mixed}}) =$$

$$p_{\text{gradual}} \cdot P(S_{t+1} = s | S_t, m_{\text{gradual}})$$

$$+ (1 - p_{\text{gradual}}) \cdot P(S_{t+1} = s | S_t, m_{\text{abrupt}}).$$
(5)

In each of these three cases, we model the probability of the first strategy as a uniform distribution over the space of decision strategies (i.e.,  $P(S_1) = \frac{1}{|S|}$ ).

Together with the observation model and the strategy space described above each of these priors defines a generative model of a participant's process tracing data **d**; this model has the following form:

$$P(\mathbf{d}) = \frac{1}{|\mathcal{S}|} \cdot \prod_{t=2}^{T} P(S_t | S_{t-1}, m) \cdot P(\mathbf{d_t} | S_t). \tag{6}$$

The three measurement models differ in the identity of  $m \in \{m_{\text{gradual}}, m_{\text{abrupt}}, m_{\text{mixed}}\}$ . Inverting these models gives rise to a computational method for measuring cognitive plasticity.

#### Computational microscopy by model inversion

The models above describe how cognitive plasticity manifests in process-tracing data. To measure cognitive plasticity we have to reason backwards from the process tracing data to cognitive changes that generated it. That is, we can build a computational microscope for measuring cognitive plasticity by inverting these measurement models. To achieve this, we leverage the Viterbi algorithm (Forney, 1973) to compute maximum a posteriori (MAP) estimates of the hidden sequence of planning strategies given the observed process tracing data, the measurement model, and its parameters ( $p_{\text{stay}}$  for  $m_{\text{abrupt}}$  and  $p_{\text{gradual}}$  and  $p_{\text{stay}}$  for  $m_{\text{mixed}}$ ). To estimate the model parameters we perform grid search with a resolution of 0.02 over  $p_{\text{stay}} \in [0,1]$  for  $m_{\text{abrupt}}$  and  $(p_{\text{stay}}, p_{\text{gradual}}) \in [0,1] \times [0,1]$  for  $m_{\text{mixed}}$ .

Inferring the hidden sequence of cognitive strategies in this way lets us look at cognitive plasticity through the lens of a computational microscope.

# Validating the computational microscope Validation on synthetic data

To validate our computational microscope, we apply it to simulated process tracing data. To avoid bias towards any one measurement model, we sampled 100 simulated trials from each of the three measurement models and combined them into a single data set comprising 300 simulated trials in total.

We then invert the three measurement models on each of the simulated trials (**d**) and compared the maximum a posteriori estimate of each strategy sequence ( $\hat{\mathbf{S}}$ ) against the ground truth (S) in terms of the proportion of correctly inferred strategies and the distance between the inferred strategies and the ground truth. To measure the distance between two sequences of n planning strategies we define  $\Delta(\mathbf{v}, \mathbf{w})$  as

 $\frac{1}{n} \cdot \sum_{i=1}^{n} \Delta(v_i, w_i)$ . For better interpretability, the relative distance  $\Delta_{\text{rel}}(s_1, s_2) = \Delta(s_1, s_2)/\bar{\Delta}$  normalizes  $\Delta(s_1, s_2)$  by the average distance between any strategy and its closest neighbour.

As a baseline, we evaluated the computational method that inverts the observation model in Equation 1 on each click sequences independently. This simple approach was sufficient to infer the correct strategy about 81% of the time (95% confidence interval: [80.2%,81.8%]). The average distance from the inferred strategy to the true one was only 21% of the average distance from each strategy to its closest neighbor ( $\Delta_{\rm rel}(\mathbf{\hat{s}}^{\rm baseline},\mathbf{s})=0.215,\,95\%$  confidence interval: [0.20,0.23]). This shows that the simulated click sequences were highly diagnostic of the strategies that generated them.

We found that exploiting the temporal dependencies among subsequent strategies by using either of the three measurement models significantly improved the proportion of correctly inferred strategies to 88.5%, 88.3%, and 88.5% for  $m_{\text{gradual}}$ ,  $m_{\text{abrupt}}$ , and  $m_{\text{mixed}}$  respectively (all p < 0.0001) and decreased the average distance between the inferred strategies and the ground truth by more than 40% ( $\Delta_{rel}(\mathbf{\hat{s}}^{gradual}, \mathbf{s}) =$ 0.124,  $\Delta_{rel}(\mathbf{\hat{s}}^{mixed}, \mathbf{s}) = 0.124$ , and  $\Delta_{rel}(\mathbf{\hat{s}}^{abrupt}, \mathbf{s}) = 0.127$ , all p < 0.0001). The minor differences between the accuracies and distances achieved with the three measurement models were not statistically significant ( $\chi^2(2) = 0.36$ , p = 0.8373and F(2,897) = 0.06, p = 0.942 respectively). These results suggest that – under reasonable, theory-agnostic assumptions about what cognitive plasticity might be like – our computational microscopes can be expected to produce more accurate measurements than simpler methods.

Which measurement model is most suitable depends on whether the measured changes are mostly gradual, mostly abrupt, or a combination of both. This may vary across tasks and participants. We therefore invert all three measurement models on each participant's data and select the most appropriate measurement model for each participant according to the Akaike Information Criterion (Akaike, 1974). We then interpret the inferences obtained from inverting the selected model as the measurement of our computational microscope.

#### Validation on empirical data

To validate our computational microscope on empirical data, we applied it to the Mouselab-MDP process-tracing data from Experiments 1–3 by Lieder (2018) where 176 participants solved 31 different 3-step planning problems of the form shown in Figure 1. Concretely, we asked if our computational microscope can detect the effect of an experimental manipulation expected to promote cognitive plasticity, namely the feedback participants in the second condition of Experiment 1 received on the (sub)optimality of their chosen actions. This feedback stated whether the chosen move was optimal and included a delay penalty whose duration was proportional to the difference between the expected returns of the chosen move versus the optimal one.

Our computational microscope successfully detected this manipulation. As shown in Figure 2, the inferred learninginduced changes were significantly larger in the feedback

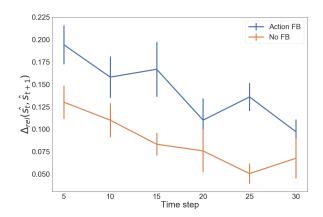


Figure 2: Feedback accelerates cognitive plasticity. This figure shows that feedback increased the amount of cognitive plasticity at the beginning of learning.

condition than in the control condition in the first 15 trials and in trials 21–25 ( $p \le 0.012$  for each 5-trial bin) and nearly significant in trials 15–20 (p = 0.08) and trials 25–30 (p = 0.06). Furthermore, Figure 2 also shows that cognitive plasticity slowed down over time as participants adapted to experiment's stationary decision environment.

Next, we performed  $\chi^2$ -tests to determine which strategy transitions were facilitated versus suppressed by the provision of action feedback. We found that action feedback selectively increased the probability of eight performance-increasing strategy changes (and only two performance decreasing ones) while decreasing the probability of five performance-decreasing transitions, five self-transitions, and only one performance-increasing transition.

Our method's ability to detect the plasticity-enhancing effects of feedback suggests that its inferences provide a valid measure of cognitive plasticity.

## Shedding light on cognitive plasticity

Having validated our computational microscope on both simulated and empirical data, we now leverage it to measure how people learn how to plan by applying it to the process tracing data from the control conditions of Experiment 1 and the training phases of the control conditions of Experiments 2 and 3 from Lieder (2018). In the following, we illustrate how our computational microscope can be used to i) measure how people's propensity to use different cognitive strategies evolves over time, ii) test theories of cognitive development and cognitive plasticity, and iii) characterize people's metacognitive learning trajectories.

**Temporal evolution of strategy frequencies.** As shown in Figure 3, we found that the most common initial strategy was to act impulsively without any planning (*No Planning*). The prevalence of this strategy decreased gradually

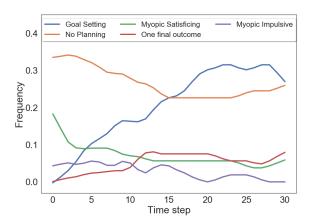


Figure 3: Time course of strategy usage frequencies of the five most common strategies.

over time from about 34% to about 26% ( $\chi^2(1) = 7.95$ , p = 0.0048)<sup>2</sup>. Conversely, the frequency of the near-optimal *Goal Setting* strategy increased from about 4% to 30% ( $\chi^2(1) = 148.85$ , p < 0.0001). The frequencies of the two maladaptive strategies that decide based on immediate rewards (*Myopic Satisficing* and *Myopic Impulsive*) dropped from about 11% and 4% respectively to about 5% ( $\chi^2(1) = 11.74$ , p = 0.0006) and 0.6% ( $\chi^2(1) = 11.62$ , p = 0.0006) respectively, whereas the frequency of the strategy *One Final Outcome* that prioritizes long-term consequences increased from about 1% to about 6% ( $\chi^2(1) = 20.22$ , p < 0.0001). Jointly these strategies accounted for about 53%–72% of our participants planning in the different trials of our experiment.

**Testing theories of cognitive development.** Prominent theories of cognitive development disagree about whether it proceeds in discrete stages (Piaget, 1971) with abrupt transitions or continuous gradual change (Siegler, 1996).

Our computational microscope suggested that cognitive plasticity includes both gradual and abrupt strategy changes. We observed that the data from  $63.0\% \pm 4.9\%$  of our participants was best captured by the abrupt model, while the data from  $29.8\% \pm 4.6\%$  of the participants were best captured by the gradual model, and the data from  $7.2\% \pm 2.6\%$  were best captured by the mixed model. A more fine-grained analysis of the individual inferred transitions revealed that the majority of strategy changes was gradual (i.e., 59.1%,  $\chi^2(1) = 56.8$ , p < 0.0001) but there was also a non-negligible percentage of abrupt changes (i.e., 40.9%). In total those different types of strategy changes constituted 22.8% of all transitions; that is 77.2% of the inferred transitions were strategy repetitions.

Siegler's overlapping waves theory (Siegler, 1996) asserts that multiple cognitive strategies are being used in parallel at each time during cognitive development. It further asserts

that the relative frequencies of these strategies shift towards increasingly more adaptive strategies and that there are intermediary strategies whose frequency waxes and vanes. To test the first prediction of the overlapping waves theory (Siegler, 1996), we performed  $\chi^2$ -tests on the strategies' frequencies in all bins of 5 consecutive trials. In support of the hypothesis that multiple different strategies are used at each point in time throughout the learning process we found that the average number of strategies used by significantly more than 5% of the participants in any given trial was 2.16 (95% confidence interval: [2.02, 2.30]). Consistent with the prediction that high-performing strategies become more prevalent over time whereas low-performing strategies become less prevalent over time we found a significant rank correlation between each strategies' average performance and the change in their frequency from the first trial to the last trial (Spearman's  $\rho(37) = 0.39$ , p = 0.0154). We did not find any evidence for intermediary strategies whose frequency initially increases and later decreases again. That is, there was no strategy whose frequency was higher in the middle two time bins than in the both the first two time bins and the last two time bins. But overall the measurements we obtained with our computational microscope are more consistent with the overlapping waves theory than with earlier stage theories of cognitive development.

**Learning trajectories.** To identify the most common learning trajectories, we categorized each inferred strategy as belonging to one of the 11 types of strategies described earlier. We then extracted the order in which different strategy types appeared in the inferred sequences. Using this analysis we found there were almost as many unique learning trajectories as there were learners: The 114 participants who changed their strategy at least once displayed 98 unique learning trajectories; that is 86.0% of the learning trajectories were unique and the remaining trajectories were exhibited by only 2–4 learners each. Zooming in on the 49 participants who learned the near optimal goal setting strategy, we found that they reached the near optimal goal setting strategy via 38 unique learning trajectories. Consistent with the overlapping waves theory we found that 83.8% of these learning trajectories included at least one intermediary strategy between the initial strategy and the final strategy. Most importantly, our analysis revealed three dominant gateways to optimal planning: 35% of the penultimate strategies inspected all potential final states – whereas the optimal strategy stops searching for better final states once it encounters the best possible outcome – and sometimes planned backwards from undesirable states; 27% of the penultimate strategies inspected the potential final states more like the optimal strategy but additionally and wastefully inspected paths towards undesirable final outcomes, and 21% of the penultimate strategies inspected both immediate and final outcomes while ignoring the intermediate states. This suggests that participants discovered the optimal goal setting strategy via intermediate strategies that per-

<sup>&</sup>lt;sup>2</sup>The  $\chi^2$ -tests in this paragraph compare the average frequency in the first five trials against the average frequency in the last five trials.

form gratuitous planning. Furthermore, we found that about 42% of participants who succeeded to learn a near-optimal goal setting strategy started with strategies that inspect both immediate and final outcomes without looking at intermediate ones. In addition to the 114 participants who changed their initial strategy, 62 participants (35%) never changed their strategy including 20% who always acted impulsively without any planning and 9% who always used frugal goal setting strategies.

More than reinforcement learning? To test the hypothesis that cognitive plasticity is partly driven by metacognitive reinforcement learning (Krueger et al., 2017; Lieder et al., 2018), we tested the effect of the absolute value of the reward prediction error in trial t (defined as reward on trial t minus the average of the rewards on trials 1 to t-1) on the subsequent change in the participant's planning strategy  $(\Delta_{\rm rel}(\hat{S}_t, \hat{S}_{t+1}))$  using an ANOVA controlling for the effect of the trial number and individual differences between participants. Surprisingly, the main effect of the reward prediction was not statistically significant (F(1,2343) = 1.13, p = 0.29) but its interaction with the participant ID (F(175, 2342) =1.42, p = 0.0003) was. To test the effect of prediction errors independently of our assumptions about the distance between strategies, we compared the average absolute values of reward prediction errors before strategy changes versus strategy repetitions while controlling for individual differences; the difference was not statistically significant (F(1, 2405) = 3.07,p = 0.08).

### **Discussion**

We have successfully validated our method on both synthetic and human data. The results suggest that our computational microscope can measure cognitive plasticity in terms of the temporal evolution of people's cognitive strategies.

Our findings suggest that this method has great potential for uncovering the mechanisms of cognitive plasticity and how they are impacted by the learning environment, individual differences, time pressure, motivation, and interventions – including feedback, instructions, and reflection prompts.

We are optimistic that computational microscopes will become useful tools for reverse-engineering the learning mechanisms that enable people to acquire complex cognitive skills and shape the way we think and decide. To make this possible, we will extend the proposed measurement model to continuous strategy spaces and learning at the timescale of individual cognitive operations.

The tentative conclusions we obtained with our first prototype of a computational microscope for measuring cognitive plasticity should be taken with a grain of salt because more psychologically plausible distance metrics and more realistic strategy representations could lead to different conclusions about the nature of cognitive plasticity. In this first step, we determined the similarity between strategies based on their behavior. But two strategies that look very different could result from similar mechanisms. Future work will identify a low-dimensional continuous strategy space by decomposing each strategy into its Pavlovian, habitual, and model-based components (van der Meer, Kurth-Nelson, & Redish, 2012). This more realistic representation will allows us to measure the similarity between strategies by comparing the underlying neurocomputational mechanisms.

The approach developed in this paper makes it possible to more directly observe the previously hidden phenomenon of cognitive plasticity in all of its facets – ranging from skill acquisition, learning to think differently, cognitive development, reflective learning, cognitive decline, self-improvement, cognitive development, changes in cognitive dispositions, and the onset, progression, and recovery from psychiatric symptoms and mental disorders. Last but not least, using this method to reverse-engineer people's ability to discover and continuously refine their own algorithms could enable substantial advances towards self-improving (general) artificial intelligence.

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